



Phase Space Formulation and Trajectory-Based Quantum Dynamics for Composite Systems with Discrete or Continuous Degrees of Freedom

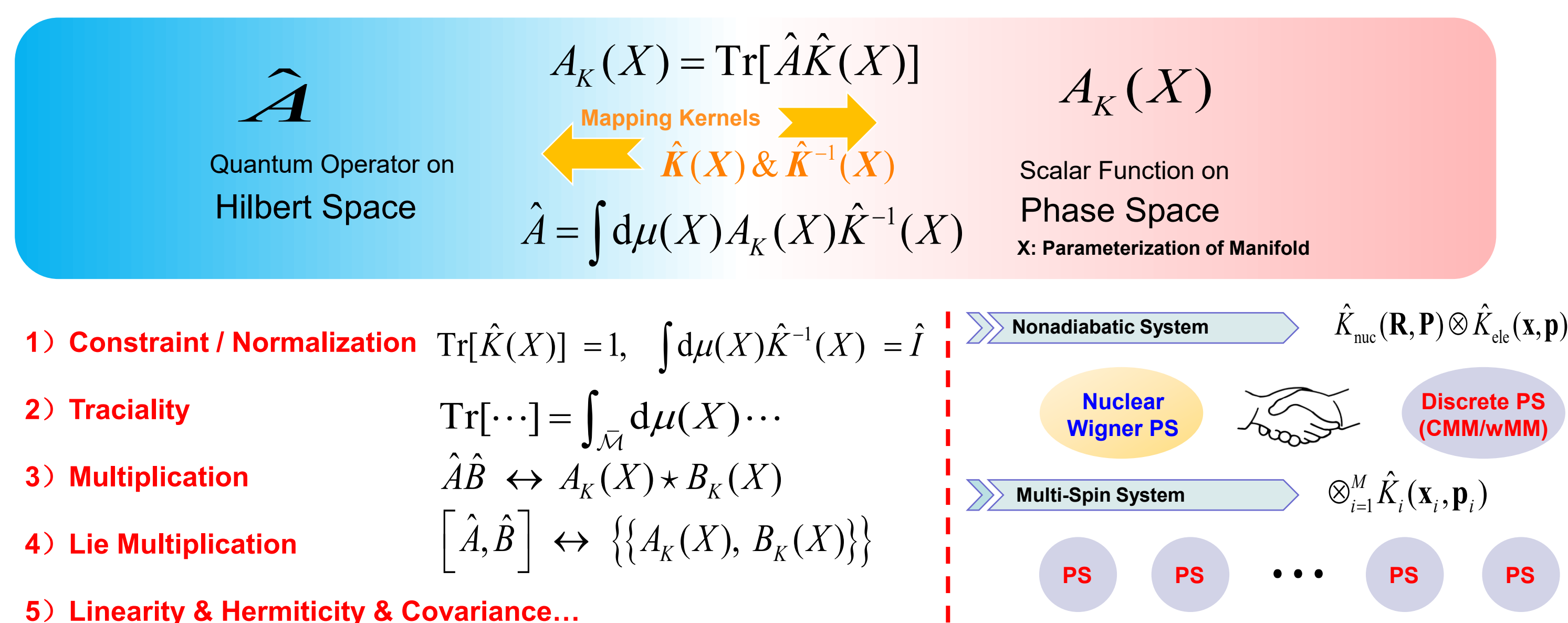
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介绍 Introduction

We have developed a new phase space formulation and a corresponding trajectory-based quantum dynamics method for composite systems, where either discrete or continuous degrees of freedom could be involved. The most essential element of the one-to-one correspondence between quantum operators and classical functions is a smooth manifold, namely, the phase space.

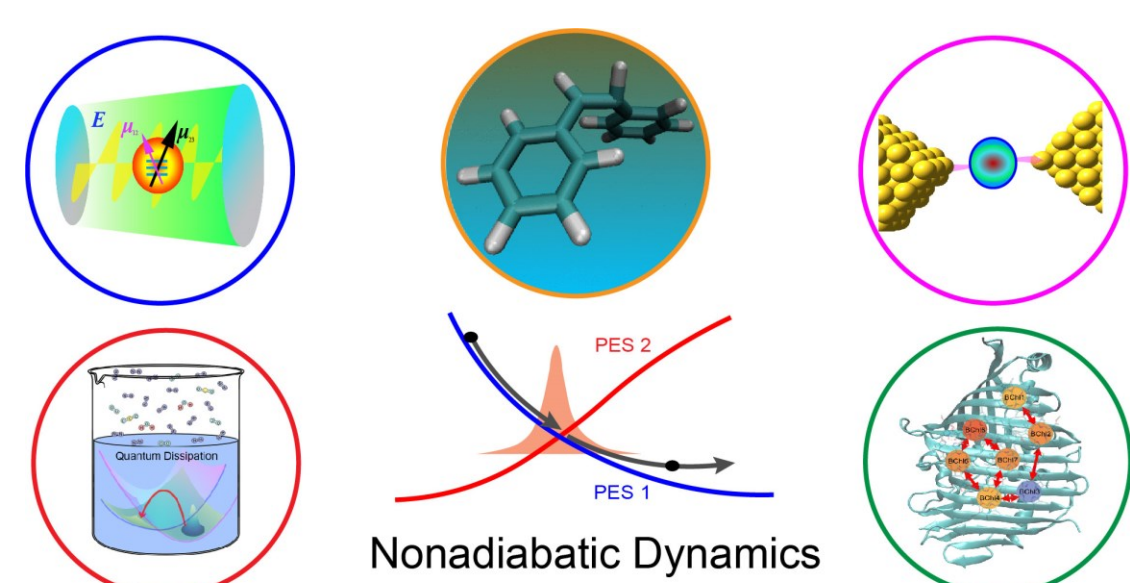


Our work first revealed the normalized constraint[1] for the discrete phase space (for electronic DOFs) and developed classical mapping models (CMM)[1-4], lying on the U(F)/U(F-1) manifold with Meyer-Miller variables, i.e., the constraint coordinated-momentum phase space (CPS). Combined with the continuous Wigner phase space for nuclear DOFs, the phase space formulation can be applied to nonadiabatic systems, as well as other composite systems. In addition, by employing characteristic function to depict a manifold and ensure the exact correspondence, we also develop weighted mapping model (wMM) based on weighted phase space (WPS).

理论框架 Theory Framework

Three key elements in the phase space framework:

1. the initial condition of the trajectory (i.e., manifold),
2. the EOMs of the trajectory, and
3. the integral expression for the expectation/ensemble.



Initial Condition

Phase space \leftrightarrow manifold

$$\int_{\mathcal{M}} F d\mathbf{x} d\mathbf{p} (\cdot) = \frac{\int F d\mathbf{x} d\mathbf{p} \mathcal{M}(\mathbf{x}, \mathbf{p}) (\cdot)}{\int d\mathbf{x} d\mathbf{p} \mathcal{M}(\mathbf{x}, \mathbf{p})}$$

$$\hat{K} = \sum_{n,m=1}^F \left[\frac{1}{2} (x^{(n)} + ip^{(n)}) (x^{(m)} - ip^{(m)}) - \gamma \delta_{nm} \right] |n\rangle \langle m|$$

Equations of Motion

$$-\partial_t \rho = \{ \{ \rho(\mathbf{R}, \mathbf{P}, \mathbf{x}, \mathbf{p}, t), H(\mathbf{R}, \mathbf{P}, \mathbf{x}, \mathbf{p}) \} \}_{\text{Moyal}}$$

$$\xrightarrow{\alpha(\hbar) \rightarrow \text{MFA}} \{ \rho(\mathbf{R}, \mathbf{P}, \mathbf{x}, \mathbf{p}, t), H(\mathbf{R}, \mathbf{P}, \mathbf{x}, \mathbf{p}) \}$$

$$H = \mathbf{P} \mathbf{M}^{-1} \mathbf{P} / 2 + \sum_{n,m=1}^F \text{Tr}[\hat{K}_{\text{elc}}(\mathbf{x} \mathbf{p}) \hat{V}(\mathbf{R})]$$

Time Correlation Function

$$\int_{\mathcal{S}} F d\mathbf{X} K_{nm}(\mathbf{X}) [K^{-1}]_{kl}(\mathbf{X}) = \delta_{mk} \delta_{nl}$$

CMM Phase Space (CPS)
 $U(F)/U(F-1)$ manifold: (\mathbf{x}, \mathbf{p})
Stratonovich Phase Space
 $SU(F)/U(F-1)$ manifold: (θ, φ)

Normalized Discrete DOFs
 $\text{Tr}[\hat{K}(X)]_{\mathcal{M}} = 1$

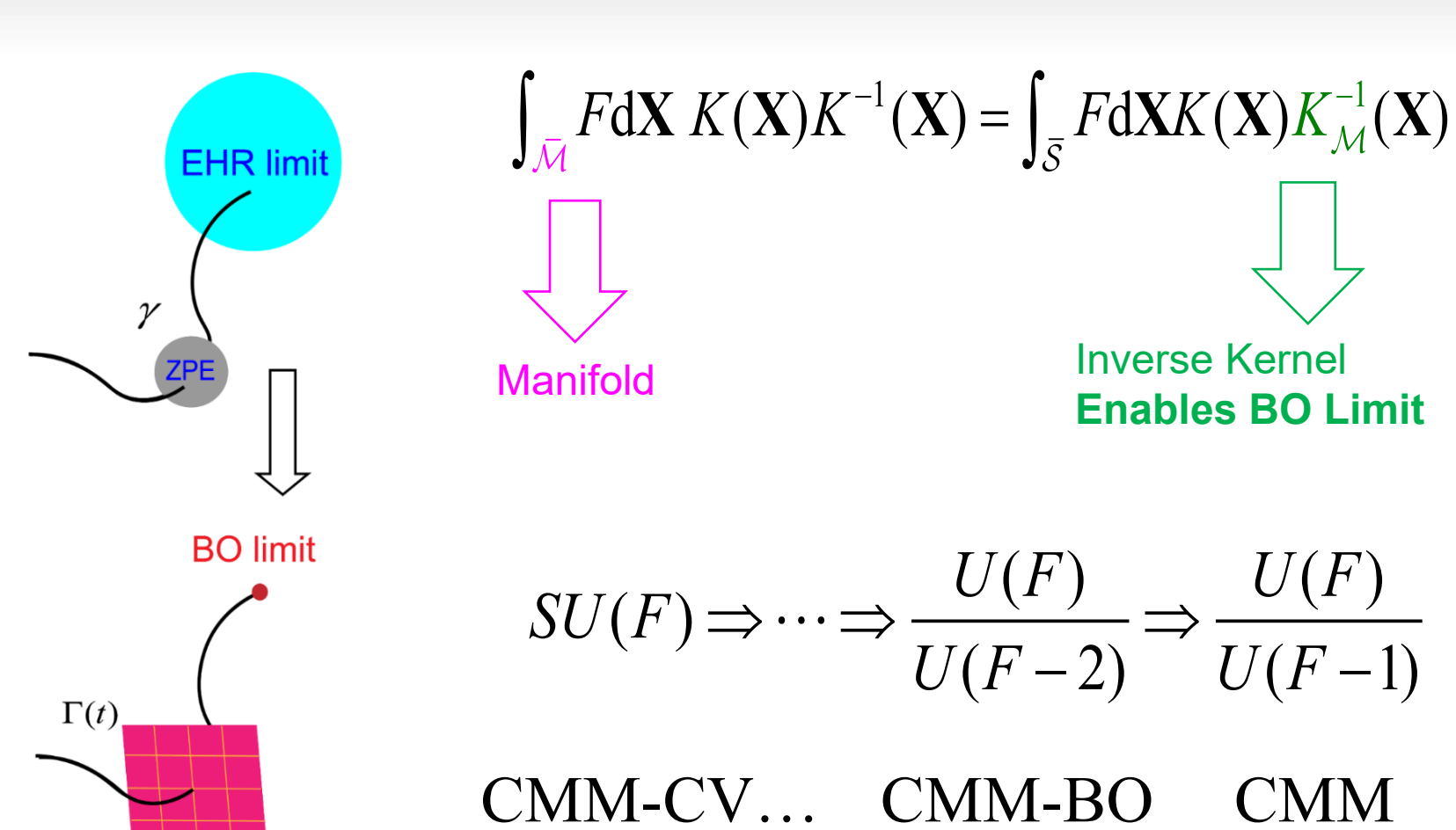
Tangent space $\mathcal{T}\mathcal{M}$

Symplectic structure generates EOMs

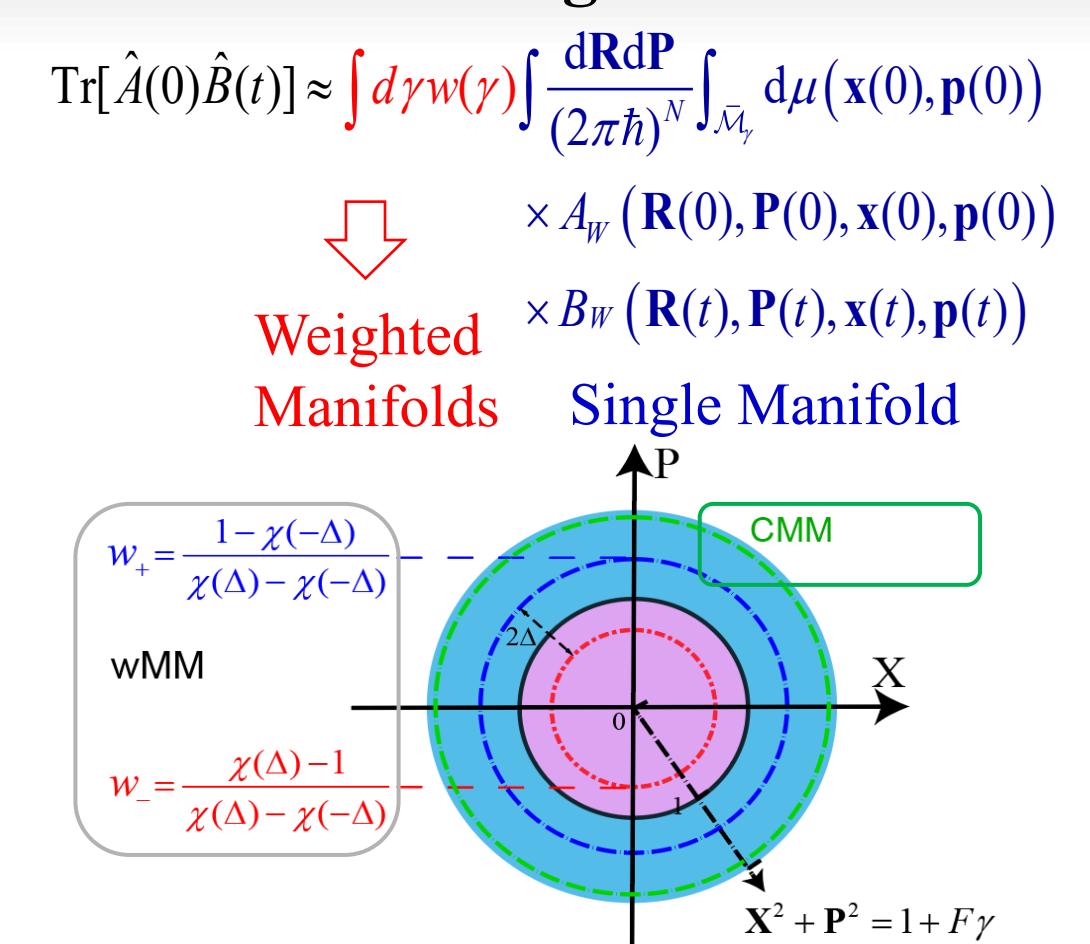
Characteristic function also shapes the manifold

$$\int d\gamma w(\gamma) \chi(\gamma) = 1$$

Generalization of Single Manifold

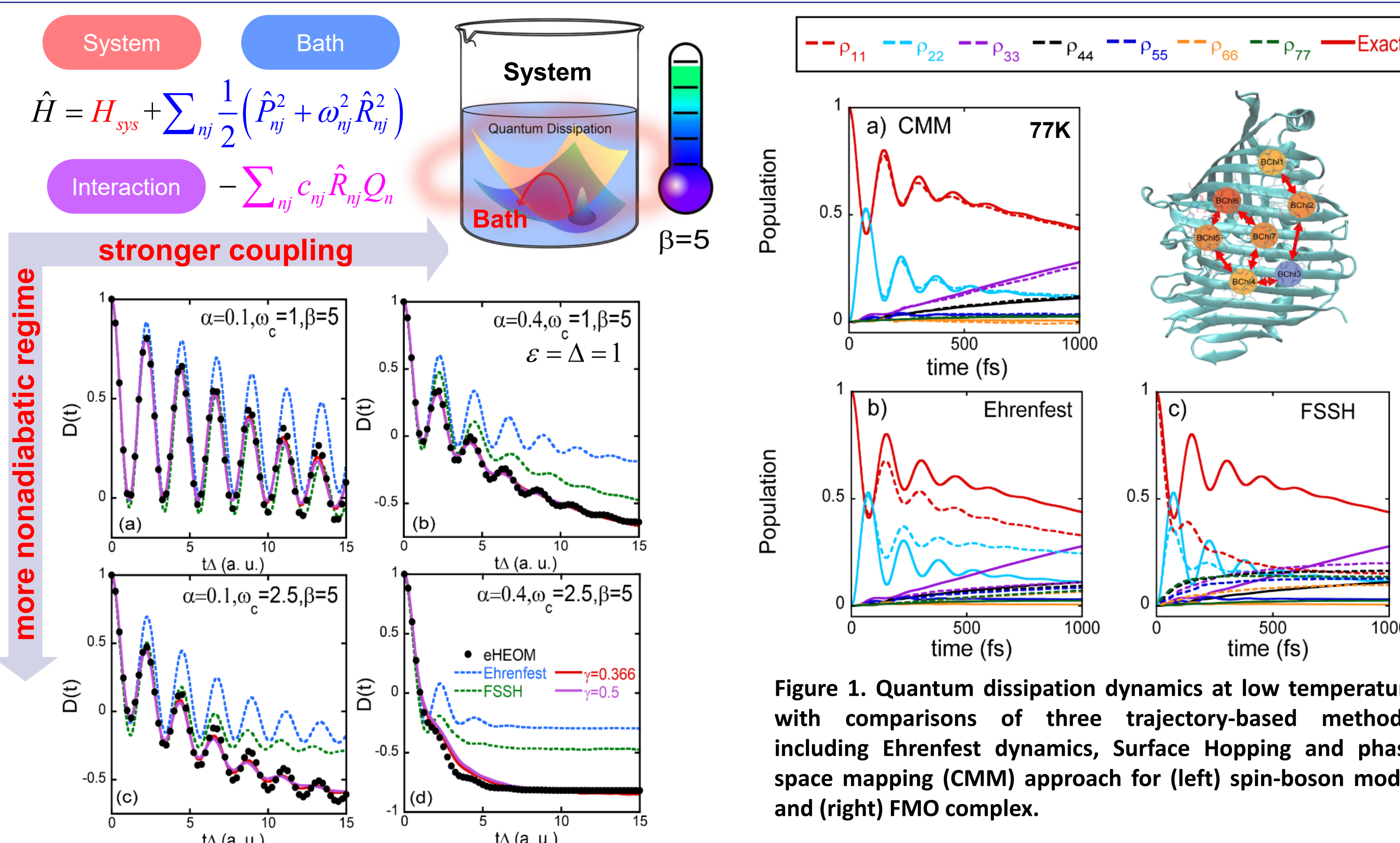


Extension to Weighted Manifolds

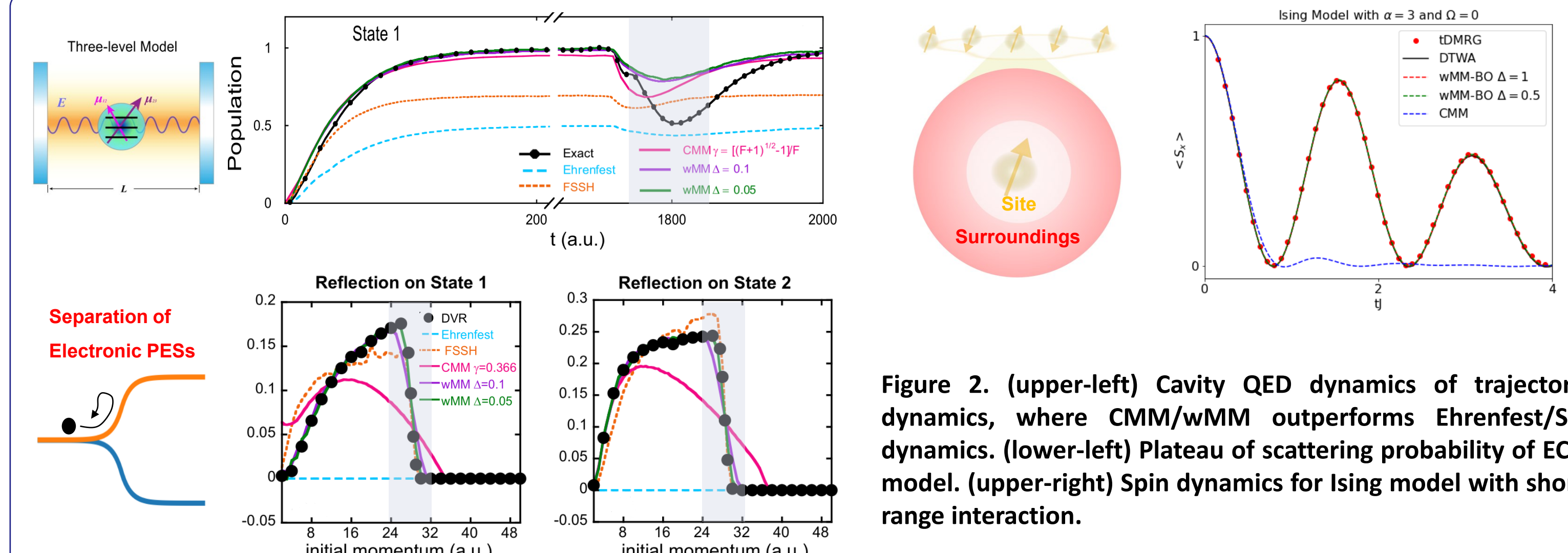


数值结果 Numerical Results

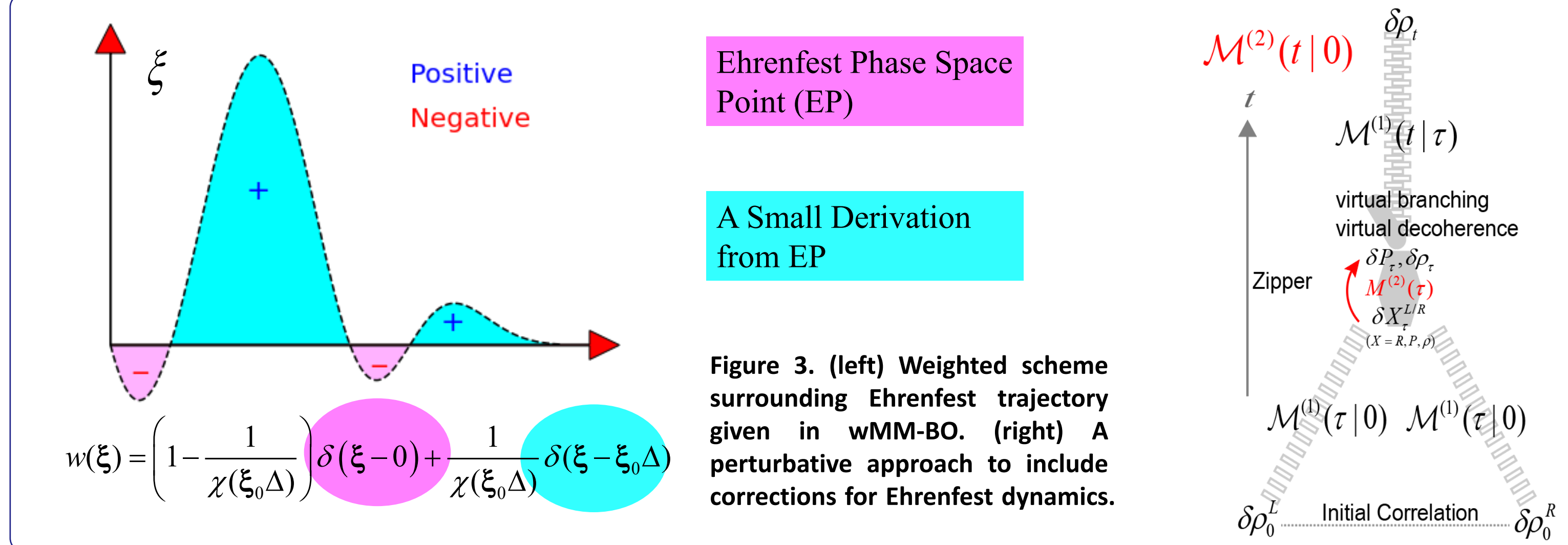
1. Constraint Coordinate-Momentum Phase Space (CMM)



2. Weighted Phase Space (wMM)



3. Weighted U(F)/U(F-2) Rank=2 Manifold Scheme: wMM-BO



结论 Conclusions

It points out a phase space family with a constraint parameter that can be negative and continuous, interpreted as a manifold shape parameter other than the so-called "ZPE" factor. We also develop generalization of single manifold and extend the mapping to weighted manifolds. We have realized different kinds of efficient mapping manifolds and mapping kernels for better description of electronic coherence and decoherence, as well as of nuclear dynamics for nonadiabatic systems. Among trajectory-based methods, phase space mapping approach exhibits advantages for composite systems.

References

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